# Planetary Radar Astronomy with Linear FM (chirp) Waveforms

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## 1 Introduction

Binary phase-coded waveforms have been used with great success in planetary radar experiments. This class of signals includes the Barker codes, pseudo-random codes, and random-long codes. Signals modulated with binary phase codes can be compressed to achieve resolution in range, and they have been used to provide images of dozens of solar system objects.

While planetary radars have used coded waveforms almost exclusively, airborne and spaceborne radars typically use linear FM (chirp) waveforms. Chirp waveforms have been studied exhaustively [1] and they may offer significant advantages compared to traditional binary phase-coded waveforms. In this document, the possibility of using a chirp radar for planetary studies is explored. Properties of the linear FM waveforms are summarized and contrasted with their phase code counterparts.

The following features of chirp radar are of particular relevance to planetary work:

- The spectrum of linear FM signals is narrowband as opposed to the sinc-type spectrum of coded waveforms. It is possible that chirp modulation will result in better klystron performance and in improved range resolution.
- Phase-coded waveforms are inherently Doppler intolerant and this results in a degraded response at non-zero frequencies. The use of chirp waveforms guarantees a clean impulse response even at large Doppler shifts.
- Chirp waveforms have intrinsically higher range sidelobes than most coded waveforms. However, they can be used in combination with weighting functions at the range compression stage, allowing a tradeoff between sidelobe level and resolution.
- Because matched filtering is accomplished digitally, the dependence on analog matched filters is eliminated.
- Special algorithms for focused imagery (high spatial resolution) or phase preservation (interferometry) are much easier to implement with chirp waveforms.

## 2 The linear FM waveform

The linear FM waveform is characterized by a transmitted signal of the form

$$s(t) = \cos(2\pi(f_0 t + Kt^2/2)), \qquad -\tau_p/2 \leq t \leq \tau_p/2, \tag{1}$$

where $f_0$ is the carrier frequency, $K$ the rate of change of frequency, and $\tau_p$ the sweep time or pulse duration. It can be seen that the frequency increases linearly as $f = f_0 + Kt$, with a total frequency excursion or modulation bandwidth equal to $K\tau_p$. An audible signal with this time-frequency behavior would sound like a chirp, hence the name commonly attributed to the waveform.

# 3 Power spectrum

The spectra of equivalent chirp and phase-coded waveforms are compared in figure 1. Both waveforms encoded the same modulating bandwidth $B$ and would provide identical range resolutions $c/2B$. Note that the transform of the chirp is essentially flat over its range of frequencies. The power is nicely concentrated within a narrow band, as opposed to the phase-coded spectrum sidelobes extending to infinity. Linear FM waveforms may therefore result in better klystron performance and may be friendlier to our spectrum neighbors.

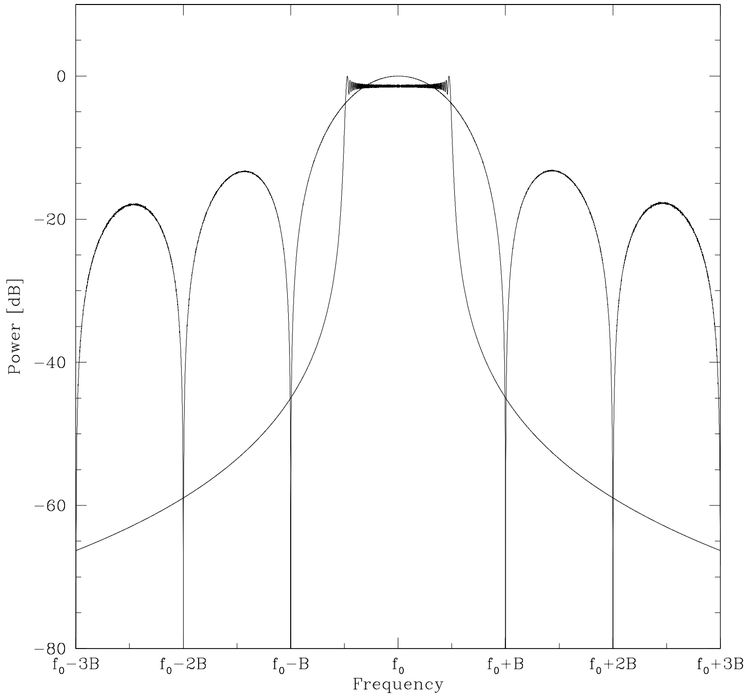

Figure 1: Sinc-type power spectrum of a binary phase-coded waveform compared to the power spectrum of a chirp waveform. The modulation bandwidth $B$ was chosen as 1/100 of the carrier frequency $f_0$. The time-bandwidth product is 1023.

In the example above, a modulation bandwidth equal to 1/100 of the carrier frequency was chosen. This ratio is comparable to what might be achieved with the Arecibo 2380 MHz planetary radar. With a maximum bandwidth of 25 MHz dictated by its klystron tubes, the Arecibo radar may be able to reach a range resolution approaching 6 meters.

# 4 Doppler tolerance

A significant advantage of chirp waveforms lies in their resilience to Doppler shifts. Figure 2 illustrates the range response of a chirp waveform with a Doppler shift equal to 0.05% of the modulation bandwidth. Note that the response is virtually unaffected, except for a time displacement. In fact, distortions to the response function only start to become noticeable at Doppler shifts equal to roughly a tenth of the modulation bandwidth [1]. The offset in time/range, a well-know feature of the linear FM ambiguity function, is given by $t_s = -f_{\rm Doppler}/K$ and results in a slight image skew. This is inconsequential since the effect can be computed and removed when converting to latitude-longitude.

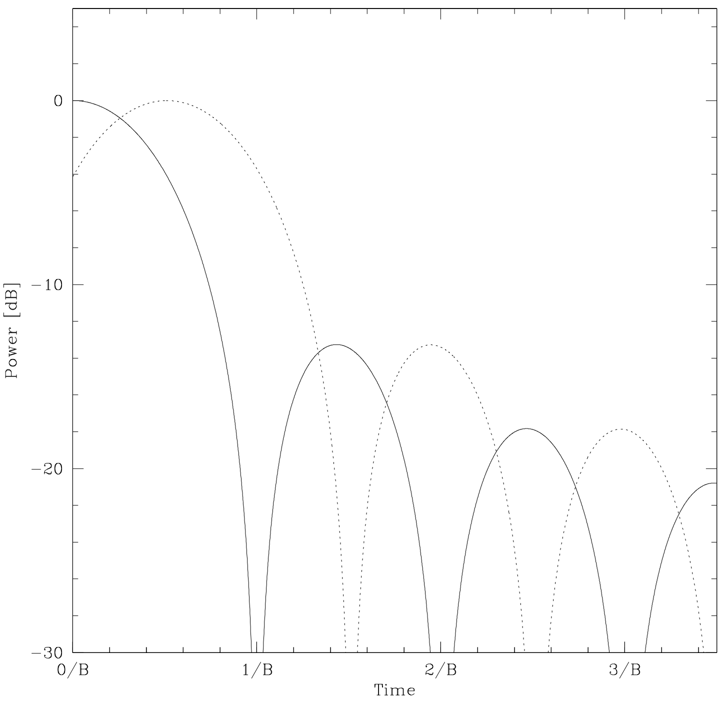

Figure 2: Range response for chirp waveforms with zero-Doppler (solid) and with a Doppler shift equivalent to 0.05% of the modulation bandwidth $B$ (dotted). The time-bandwidth product is 1023.

The response shown above would correspond to a Doppler shift of 50 Hz for a modulation bandwidth of 100 KHz, as might be the case when imaging Mercury at S-band at 1.5 km resolution. The equivalent phase-coded waveform, a 1023 code with 10 $\mu$s baud, would suffer very significant degradation in its response at such Doppler shifts. The cyclic correlation would yield sidelobe levels at -30 dB as opposed to the zero Doppler frequency level of -60 dB.

## 5 Range sidelobes

The previous section illustrated the fact that chirp waveforms do not have inherently good range sidelobe performance, with the first sidelobe only 13 dB down from the main lobe. However, the use of weighting functions at the range compression stage allows a dramatic reduction of those sidelobes, at a slight cost in resolution and SNR.

## 6 Focused and phase-preserving imaging

## 7 Implementation

Initial tests were performed with an HP 83752A frequency synthesizer on loan from Agilent Technologies. The synthesizer gave us the ability to check the response of the transmitter to chirp waveforms. During some engineering time at Arecibo on June 23, 2000, chirp signals of 1 to 25 MHz modulation bandwidth were transmitted for several minutes with the S-band planetary radar. According to our transmitter engineer, the system was much better behaved than it would have been with equivalent phase-coded waveforms.

However, the synthesizer was unable to generate the desired chirp waveforms for imaging Mercury, a target that had been selected because it has been well characterized using phase-coded techniques with the Arecibo radar [2]. We expected to use a modulation bandwidth of 100 KHz and sweep times of 16.5 ms, slightly larger than the 16.3 ms delay depth of the planet. The HP 83752A had such long retrace times between individual sweeps ($\sim$30 ms) that the experiment could not be performed. RF power also appeared at the start and stop frequencies for several milliseconds before and after the frequency ramp, which is unacceptable for radar imaging applications.

Analog Devices suggests using direct digital synthesis (DDS) to generate the linear FM signals.

The AD9854 can digitally synthesize waveforms from DC to 150 MHz with output via 12-bit I and Q digital-to-analog converters. The internal clock rate is 300 MHz which can be locked to an external reference anywhere between 5 and 300 MHz.

The chip has several modes of operation, one of which is a chirp mode ideal for chirp radar applications. A start frequency and frequency increment are loaded in 48-bit registers (1 microHz tuning steps). The frequency is incremented by the programmed amount on each transition of a ramp rate clock, wich can range from 150 Hz to 150 MHz (The ramp rate clock is based on a 20-bit divider of the internal clock). The synthesized frequency therefore linearly increases (or decreases) with time.

The start/stop of each chirp is triggered by an update signal, which can be either user-supplied or internally generated. The specifications claim that this allows for practically instantaneous return to the beginning frequency (essentially one 300 MHz clock cycle). The update signal, which can take place up to 100 million times a second, is also used to reprogram the start frequency and frequency increment in the 48-bit registers. This implies that chirp parameters could be adjusted rapidly if need be.

As an example, if we wanted to achieve a 10 ms chirp with a modulation bandwidth of 30 MHz (assuming 70 MHz center frequency), one would program the start frequency at 55 MHz, the frequency increment at 20 Hz, and set the ramp rate clock at 150 MHz. An update signal every 10 ms would then restart the chirp continuously.

An evaluation board + chip was bought for $250. This device performs extremely well, but the AD9854 evaluation software does not allow resetting the phase accumulator at every pulse. It is therefore not suitable for chirp waveforms without additional hardware modifications.

## References

[1] Charles E. Cook and Marvin Bernfeld. *Radar Signals*. Artech House, 1993.

[2] J. K. Harmon, P. J. Perillat, and M. A. Slade. High-resolution radar imaging of Mercury's North Pole. *Submitted to Icarus*, 2000.